Highlights 2021
DNRF Chair VIVEK SHENDE

My DNRF chair grant began in August 2021. In the subsequent four months, my research was centered around three main topics.

1) **Mirror symmetry and geometric Langlands.** The Langlands program originates in number theory (e.g. it was crucial to the solution of the Fermat problem about the equation $a^n + b^n = c^n$), and its 'geometric' variant has long been known to have various links to various subjects in theoretical physics, most recently, the ideas around homological mirror symmetry from the string theory literature. A central question in geometric Langlands is the construction of what are called 'Hecke eigensheaves'. It is known how to do this in some limited cases but not in general. In the preprint "Microsheaves from Hitchin fibers via Floer theory" (https://arxiv.org/abs/2108.13571), I explain how my previous results with Nadler, Ganatra, and Pardon can be used to give candidates for many new eigensheaves, and I check some expected properties of these. In work-in-progress with Nadler, I am continuing to investigate related questions.

2) **Mirror symmetry near large volume.** Benjamin Gammage and I recently proved a result which establishes homological mirror symmetry "at large volume", i.e., for certain rather degenerate spaces. In fact it is expected that this result extends to spaces "near large volume". An analogy: there is a certain function f, we calculated already its value at 0, and now we want write the Taylor series expansion around 0. Our next task is to develop the geometric methods needed to do so; specifically, to show that a space X "near large volume"admits an explicit decomposition $X = X_{lvl} \cup X_{\infty}$, where X_{lvl} is the "large volume" space we have already studied, and X_{∞} is what is called a "simple normal crossings divisor". This is expected to be possible but not yet done in the literature. The simplest case is when X is what is called an abelian variety; Gammage and I are in the process of trying to write down a construction in this setting. We also spent some time in fall identifying what difficulties we would face in the more general case.

3) **Foundations of higher genus open curve counting.** Previously, Tobias Ekholm and I wrote some articles about counting higher genus holomorphic curves with Lagrangian boundary in Calabi-Yau 3-folds. The first of these articles was mathematically complete. The next required on some (explicitly) conjectural foundational work in the subject around the existence of perturbations for the holomorphic curve equation satisfying certain explicit conjectural properties. Now we are working on constructing these perturbations.

The DNRF chair also funds two postdocs: Ikshu Neithalath and Daria Poliakova. Daria has been working on certain structural questions about "A1 categories" (the fundamental structures on one side of homological mirror symmetry). Ikshu has been studying a 4-dimensional "skein module", which, among many other applications, may be expected to play a role in a future development of my work with Tobias Ekholm (Uppsala).