

Highlights 2022

DNRF Chair VIVEK SHENDE

This DNRF chair grant began in August 2021. The work in the calendar year 2021-2022 was largely organized around the following three articles, now available on the arxiv:

- 1) **Perverse microsheaves** (with Laurent Cote, Chris Kuo, David Nadler; <https://arxiv.org/abs/2209.12998>). The Riemann-Hilbert correspondence concerns the question ‘which sheaves arise as sheaves of solutions to differential equations’, and was famously solved in the 80s. One outcome of this and related ideas was the discovery of the ‘perverse t-structure’ on sheaves, which has played an absolutely fundamental role in geometric representation theory and algebraic geometry since then. In the context of Vivek’s work on microsheaf theory and Fukaya categories, it is natural to look for an analogous t-structure in this setting. This is what is constructed in the above paper. The next step in this direction is to prove the analogous Riemann-Hilbert theorem in this setting; this work has been started.
- 2) **Invariance of microsheaves on stable Higgs bundles** (with David Nadler; <https://arxiv.org/abs/2301.01342>). The Geometric Langlands conjecture is a central question of modern mathematics; it asserts that two different categories are equivalent. Nadler and Ben Zvi recently introduced a variant, the ‘Betti’ Geometric Langlands conjecture. This variant has the feature that one of the two sides is visibly invariant under certain deformations of parameters, while the other side is not visibly so; thus the conjecture predicts said invariance. The purpose of the above paper is that it identifies a closely related category where the invariance can be shown to hold.
- 3) **Ghost bubble censorship** (with Tobias Ekhholm; <https://arxiv.org/abs/2212.05835>). It is a classical fact of algebraic geometry that the topology of a limit of a family of smooth Riemann surfaces can only be different from the general family member if the limit is singular. The purpose of the present article is to show that some qualitatively similar feature holds for a certain class of perturbations of the holomorphic curve equation. This is essential to our ongoing work on skein valued holomorphic curve counting because for this purpose we must perturb the equation to arrive in a sufficiently generic situation, but must also exclude the possibility of topology change.

The DNRF chair also funds postdocs such as: Ikshu Neithalath and Daria Poliakova, and a graduate student Zhongyu Zhang. Daria has been studying combinatorial aspects of the spaces which organize the algebra of mirror symmetry. Ikshu has been studying a 4-dimensional “skein module”, which may be expected to play a role in a future development of my work with Ekhholm (Uppsala). Zhongyu is studying stability conditions on Fukaya categories; I hope to eventually relate this to the work on perverse microsheaves mentioned above.